

Uncertainty Quantification for Complex Models

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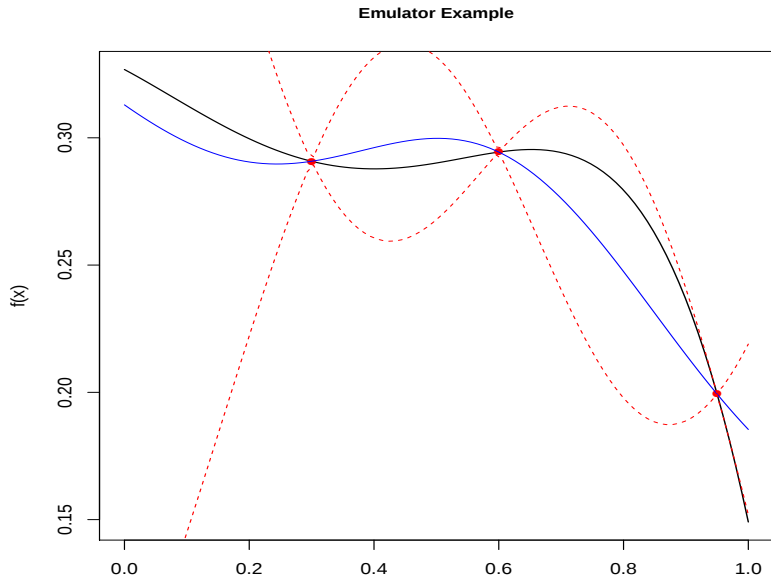
Emulation

- In order to search the parameter space for such expensive models we build **emulators**.
- An emulator is simply a statistical model for a complex function $f(x)$.
- For any x we can obtain a prediction and express our uncertainty on that prediction.
- A typical form for an emulator for output i of $f(x)$ is

$$f_i(x) = \sum_{ij} \beta_{ij} g_j(x) \oplus \epsilon_i(x)$$

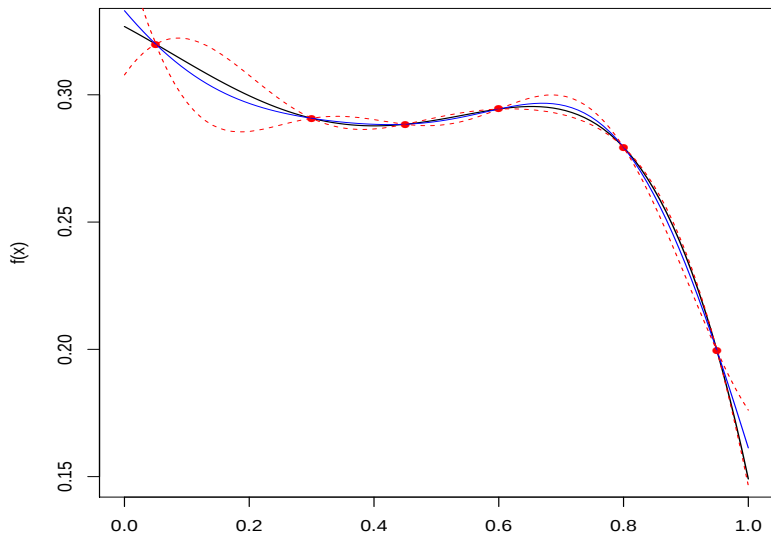
where $g(x)$ is a vector of regressors and $\epsilon_i(x) \sim \text{GP}(0, c(\cdot, \cdot; \psi))$.

Emulators



Emulators

Emulator Example



Geometric and Probabilistic Calibration

Probabilistic calibration or Bayesian calibration:

$$y = f(x^*) \oplus \eta, \quad z = y \oplus e$$

$$P(x^*|z) \propto P(x^*)P(z|x^*)$$

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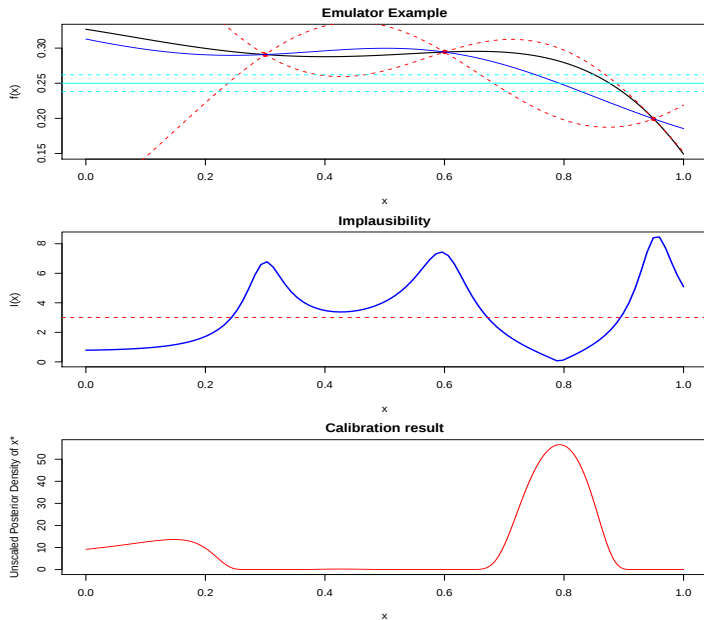
Geometric Calibration or history matching:

$$\mathcal{I}_i(x)^2 = \frac{(z_i - E[f_i(x)])^2}{\text{Var}[z_i - E[f_i(x)]]}.$$

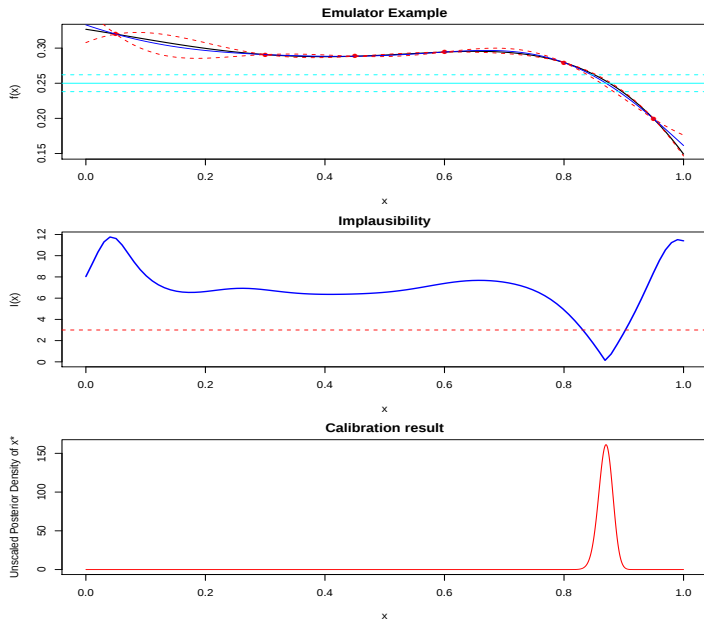
Implausibility, $\mathcal{I}(x) = \max_i |\mathcal{I}_i(x)|$.

A point x_0 is ruled out of parameter space if $|\mathcal{I}(x_0)| > a$ for some threshold a .

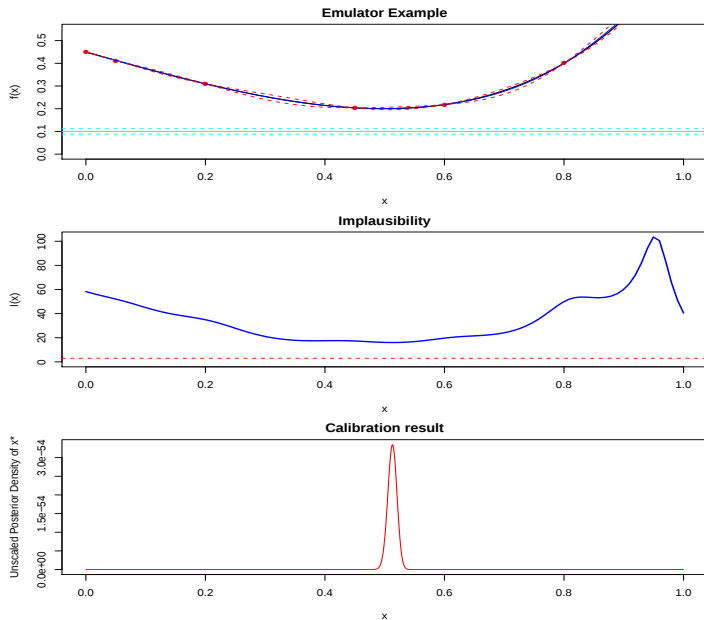
The hunt for useful models



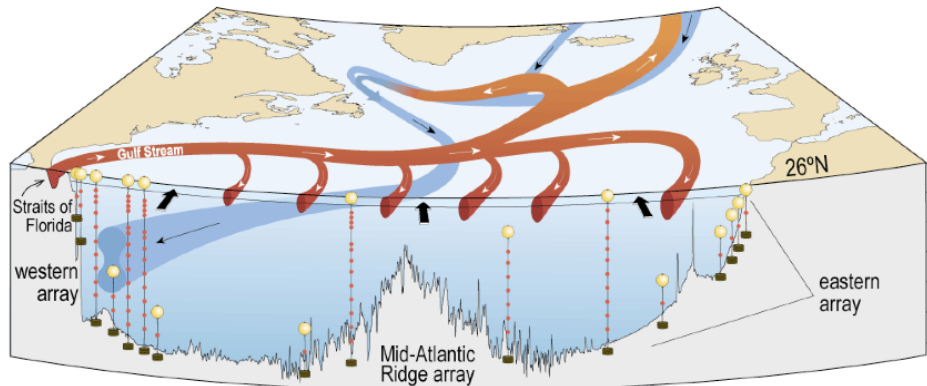
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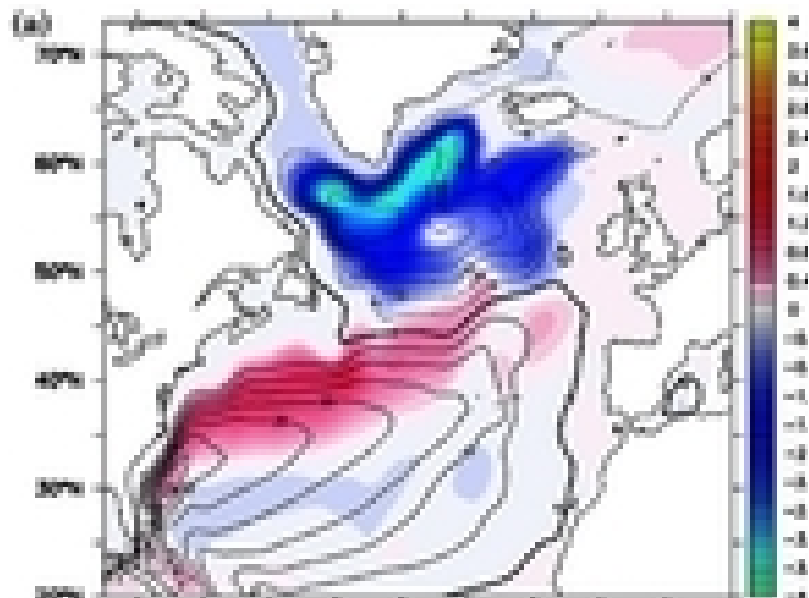
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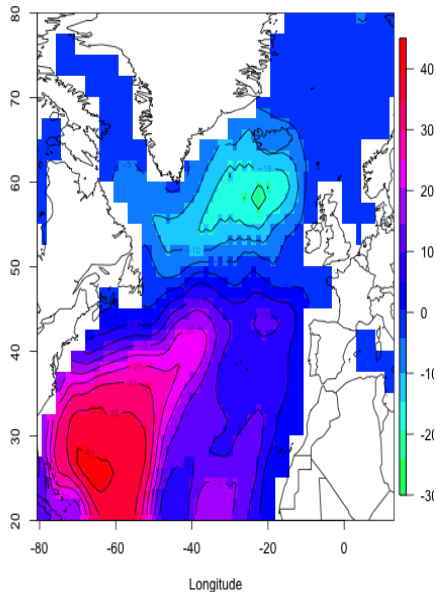
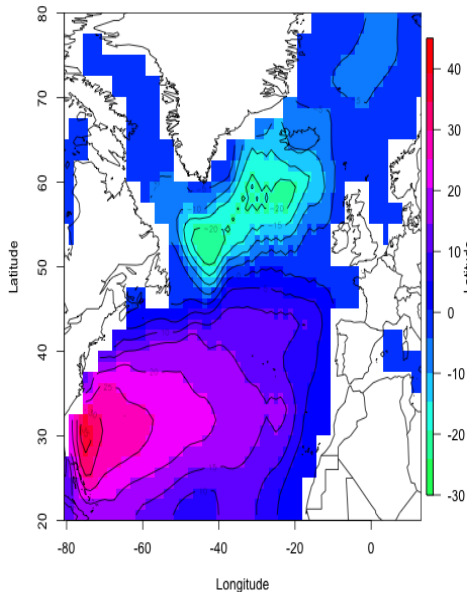
Ocean circulation 101



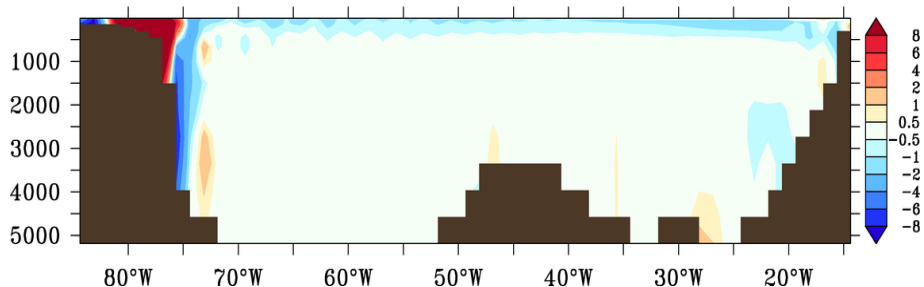
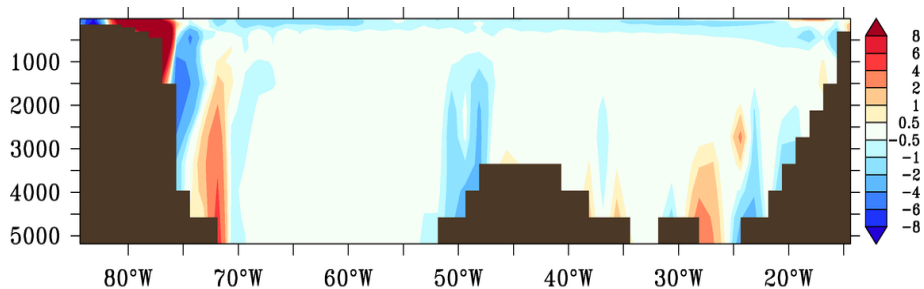
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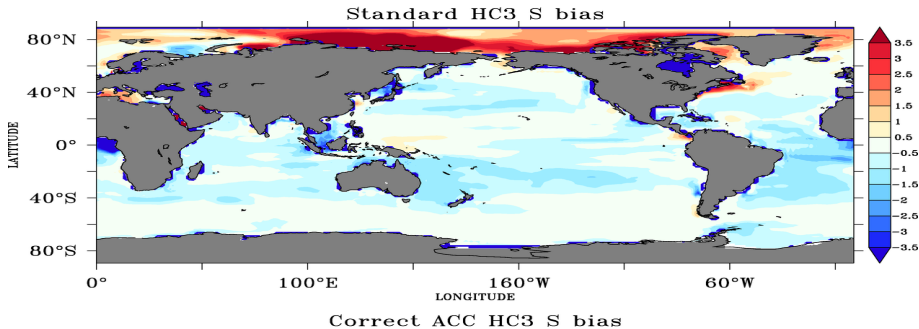
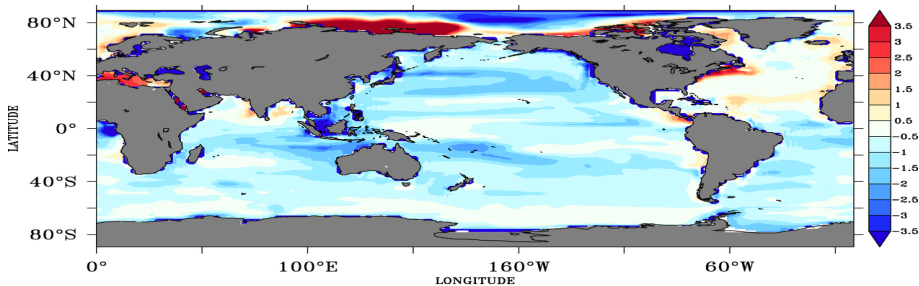
The Gyres



AMOC cross section



Global Sea Surface Salinity Anomalies



Global Precipitation Anomalies

