Some result related to intrinsic Diophantine approximation

In the past ten years, following a seminal work of Fishman and Simmons [2], the topic understanding the rational approximation inside self-similar fractals at "intrinsic" speed has raised many interests. In particular in the case of the middle-third Cantor set $K_{1/3}$, in [3], Tan, Wang and Wu established a metric description of the sets

$$E_{\psi,\text{int}} = \left\{x \in K_{1/3} : \ |x-r| \leq \psi(h_{\text{int}}(r)) \text{ f.i.m. } r \in \mathbb{Q} \cap K_{1/3}\right\},$$

where $h_{\text{int}}(r)$ denotes the intrinsic height, introduced in [2], $\psi : \mathbb{N} \to \mathbb{R}_+$ is a non increasing mapping and "f.i.m." means " for infinitely many". Regarding the more general of self-similar fractals associated with IFS's $S = \left\{f_i(x) = \frac{x}{q_i} + \frac{p_i}{q_i}\right\}_{1 \leq i \leq m}$, where $q_i \in \mathbb{N}$ and $p_i \in \mathbb{Z}$, in [1], Baker obtained partial results related to the measure (for a natural measure) of the sets

$$E_{\psi,\text{int}} = \{x \in K_S : \ |x-r| \leq \psi(h_{\text{int}}(r)) \text{ f.i.m. } r \in \mathbb{Q} \cap K_S\},$$

where K_S is the attractor of S. I will present an extension to the result of [3] to any self-similar IFS of the form $S = \left\{f_i(x) = \frac{x}{q_i} + \frac{p_i}{q_i}\right\}_{1 \leq i \leq m}$, by showing that for any non increasing $\psi : \mathbb{N} \to \mathbb{R}_+$, one has

$$\dim_H E_{\psi,\text{int}} = \frac{\dim_H K_S}{\max\left\{1, \liminf_{n \to +\infty} \frac{-\log \psi(q)}{\log q}\right\}}.$$

References

[1] S. Baker. Intrinsic Diophantine approximation for overlapping iterated function systems. Math.Ann., page 31 pp., 2022.
[2] S. Fishman and D. Simmons. Intrinsic approximation for fractals defined by rational iterated function systems: Mahler's research suggestion. Proc.Lond.Math.Soc., 109, 2014.
[3] B. Tan, B. Wang, and J. Wu. Mahler's question for intrinsic diophantine approximation on triadic Cantor set: the divergence theory. Math. Z., 2021.